B Sc General

PHYSICS

Semester-IV

DSC-1D

Duration- 1 hr

Date:04/04/2020

Time- 9AM-10AM

Fourier’s Theorem

- Motivation
- Introduction
- Statement

Joseph Fourier, our hero

Joseph Fourier, 21 March 1768–16 May 1830. (*By permission of the Bibliothèque Municipale de Grenoble.*)

Fourier was obsessed with the physics of heat and developed the Fourier series and transform to model heat-flow problems.

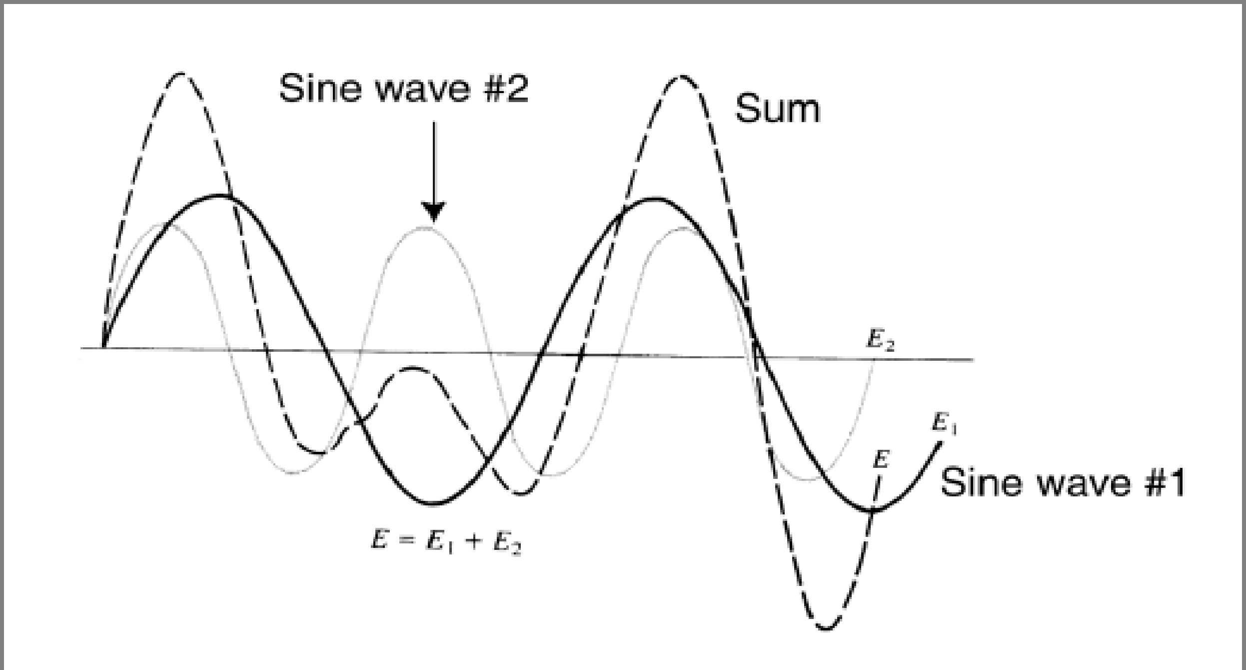
Lord Kelvin on Fourier's theorem

Fourier's theorem is not only one of the most beautiful results of modern analysis, but it may be said to furnish an indispensable instrument in the treatment of nearly every recondite question in modern physics.

Lord Kelvin

Anharmonic waves are sums of sinusoids

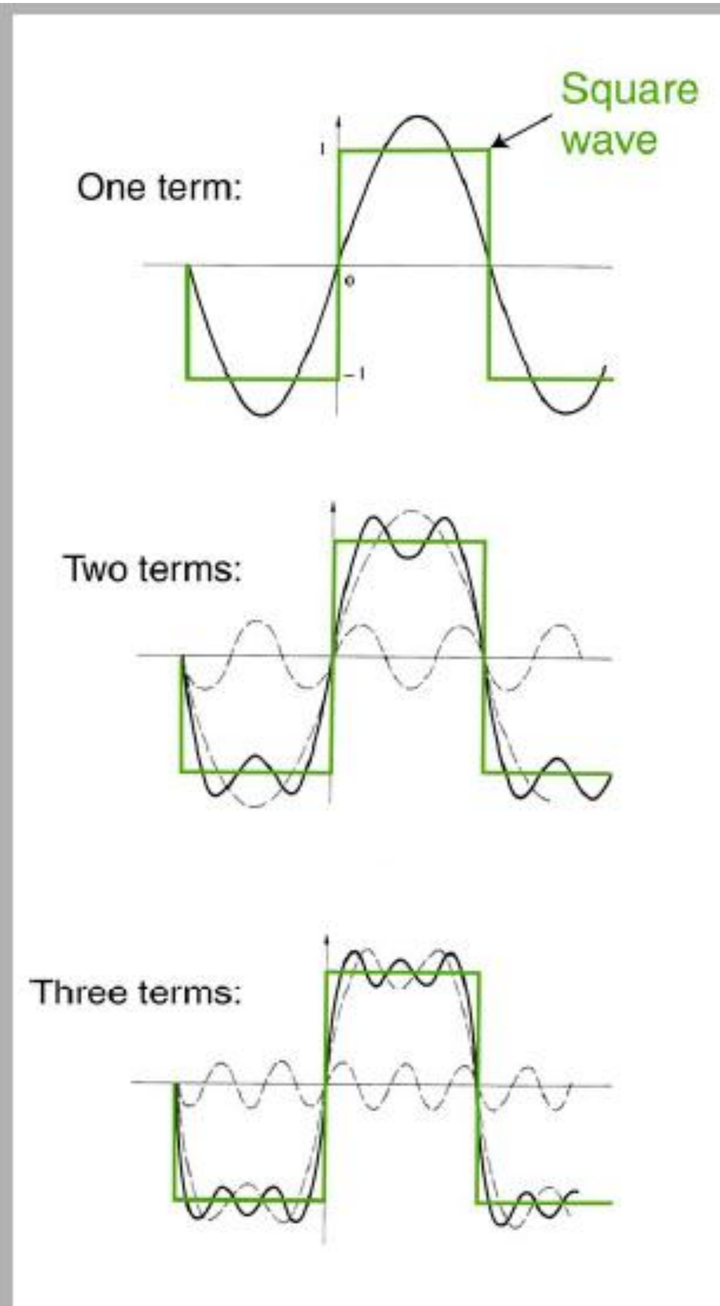
Consider the sum of two sine waves (i.e., harmonic waves) of different frequencies:



The resulting wave is periodic, but not harmonic.
Essentially all waves are anharmonic.

Fourier decomposing functions

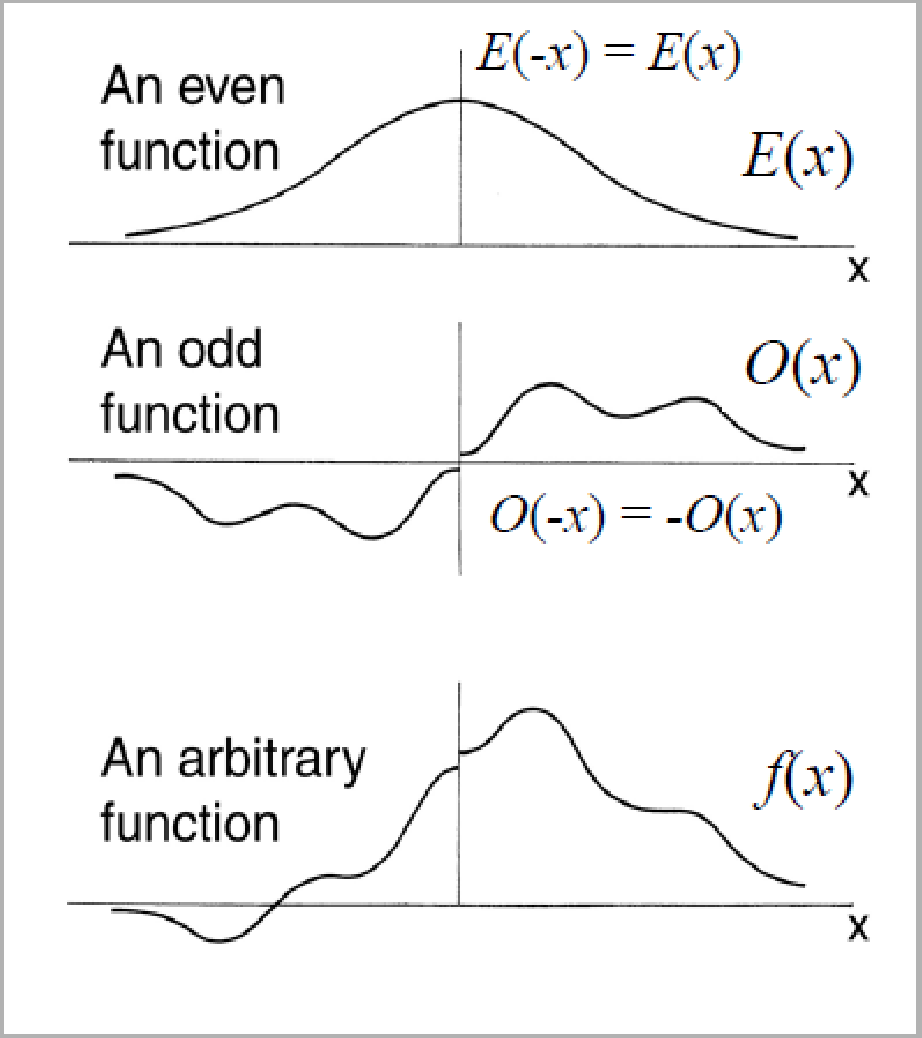
Here, we write a square wave as a sum of sine waves.



The Kronecker delta function

$$\delta_{m,n} \equiv \begin{cases} 1 \text{ if } m = n \\ 0 \text{ if } m \neq n \end{cases}$$

Any function can be written as the sum of an even and an odd function



Let $f(x)$ be any function.

$$E(x) \equiv [f(x) + f(-x)]/2$$

$$O(x) \equiv [f(x) - f(-x)]/2$$

$$\Downarrow$$

$$f(x) = E(x) + O(x)$$

Fourier Cosine Series

Because $\cos(mt)$ is an even function (for all m), we can write an even function, $f(t)$, as:

$$f(t) = \frac{1}{\pi} \sum_{m=0}^{\infty} F_m \cos(mt)$$

where the set $\{F_m;\, m = 0, 1, \ldots\}$ is a set of coefficients that define the series.

And where we'll only worry about the function $f(t)$ over the interval $(-\pi,\pi)$.

Fourier Sine Series

Because $\sin(mt)$ is an odd function (for all m), we can write any odd function, $f(t)$, as:

$$f(t) = \frac{1}{\pi} \sum_{m=0}^{\infty} F'_m \sin(mt)$$

where the set $\{F'_m;\ m = 0, 1, \ldots\}$ is a set of coefficients that define the series.

where we'll only worry about the function $f(t)$ over the interval $(-\pi,\pi)$.