

Study Material

B Sc General
PHYSICS
Semester-IV

DSC-1D

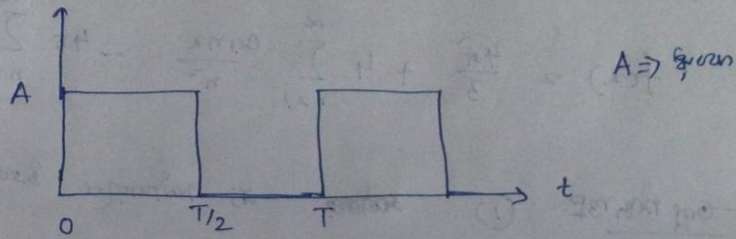
Duration- 1.30 hr

Date: 07/04/2020

Time- 7.30AM-9AM

Topic: Fourier's theorem –square wave analysis

Ex 2: Find the average value and RMS value (and Fourier series) of the following periodic waveform.



⇒ The given wave is a periodic square wave.

$$f(t) = y(\omega t) = A = \text{square}, \text{ where } 0 \leq t \leq T/2$$

$$= 0, \text{ where } T/2 \leq t \leq T$$

Fourier series expansion of the given square wave -

$$f(t) = y(\omega t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t) \quad \text{--- (1)}$$

where $a_0 = \frac{1}{T} \int_0^T f(t) dt$

$$= \frac{1}{T} \left[\int_0^{T/2} f(t) dt + \int_{T/2}^T f(t) dt \right]$$

$$= \frac{1}{T} \left[\int_0^{T/2} A dt + 0 \right]$$

$$= \frac{1}{T} \cdot \frac{T}{2} \cdot A = \frac{A}{2}$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos n\omega t dt$$

$$= \frac{2}{T} \left[\int_0^{T/2} A \cos n\omega t dt + \int_{T/2}^T 0 \cdot \sin n\omega t dt \right]$$

$$= \frac{2A}{T} \left[\frac{\sin n\omega t}{n\omega} \Big|_0^{T/2} \right]$$

$$= \frac{2A}{n \cdot \omega T} \left[\sin n\omega \frac{T}{2} - \sin 0 \right]$$

$$= \frac{2A}{n \cdot 2\pi} \left[\sin n \cdot \pi - \sin 0 \right] = 0 \quad \left[\because \omega = \frac{2\pi}{T} \right]$$

$$\begin{aligned}
 b_n &= \frac{2}{T} \left[\int_0^T y \sin n\omega t \, dt \right] \\
 &= \frac{2}{T} \left[\int_0^{T/2} A \sin n\omega t \, dt + \int_{T/2}^T 0 \cdot \sin n\omega t \, dt \right] \\
 &= \frac{2A}{T} \left[\int_0^{T/2} \sin n\omega t \, dt \right] \\
 &= \frac{2A}{T} \left[-\frac{\cos n\omega t}{n\omega} \Big|_0^{T/2} \right] \quad \left[\begin{array}{l} \omega = \frac{2\pi}{T} \\ \omega T = 2\pi \end{array} \right] \\
 &= -\frac{2A}{n\omega T} \left[\cos \frac{n\omega T}{2} - \cos 0 \right] \\
 &= \frac{A}{n\pi} [1 - \cos n\pi]
 \end{aligned}$$

n गैर-सम संख्या $n = 1, 3, 5, 7, \dots$, $\cos n\pi = -1$

$$\therefore b_n = \frac{2A}{n\pi}$$

$$\therefore b_1 = \frac{2A}{\pi}, \quad b_3 = \frac{2A}{3\pi}, \quad b_5 = \frac{2A}{5\pi}, \dots$$

n सम संख्या $n = 2, 4, 6, \dots$, $\cos n\pi = +1$

$$\therefore b_n = 0$$

$$\left(\because a_0 = \frac{A}{2} \right)$$

$$\left(\because a_n = 0 \right)$$

$$\therefore y = \frac{A}{2} + \sum_n b_n \sin n\omega t$$

$$= \frac{A}{2} + (b_1 \sin \omega t + b_3 \sin 3\omega t + \dots)$$

$$= \frac{A}{2} + \left(\frac{2A}{\pi} \sin \omega t + \frac{2A}{3\pi} \sin 3\omega t + \dots \right)$$

$$= \frac{A}{2} + \frac{2A}{\pi} \left[\frac{\sin \omega t}{1} + \frac{\sin 3\omega t}{3} + \dots \right]$$

$$= \frac{A}{2} + \frac{2A}{\pi} \sum_{m=0}^{\infty} \frac{\sin (2m+1)\omega t}{(2m+1)}$$

$$\therefore y = \frac{A}{2} + \frac{2A}{\pi} \sum_{m=0}^{\infty} \frac{\sin(2m+1)\omega t}{(2m+1)}$$

- average value of a sine wave is $\frac{2A}{\pi}$
 - average value of a square wave is $\frac{A}{2}$
 - average value of a triangular wave is $\frac{2A}{3}$
 - average value of a parabolic wave is $\frac{3A}{8}$
 - average value of a sinusoidal wave is $\frac{2A}{\pi}$

$$\left[\frac{A \sin \omega t}{\omega} - \frac{A \sin 3\omega t}{3\omega} + \frac{A \sin 5\omega t}{5\omega} - \dots \right]$$

$$\left[\frac{A}{\pi} - \frac{A}{2\pi} \cos 2\omega t + \frac{A}{4\pi} \cos 4\omega t - \dots \right]$$

$$\frac{A \sin \omega t}{\omega} - \frac{A \sin 3\omega t}{3\omega} + \frac{A \sin 5\omega t}{5\omega} - \dots$$

$$\frac{A}{\pi} - \frac{A}{2\pi} \cos 2\omega t + \frac{A}{4\pi} \cos 4\omega t - \dots$$

$$\frac{A}{\pi} + \frac{A}{\pi} \sum_{n=1}^{\infty} \frac{\cos 2n\omega t}{n^2}$$

$$\frac{A}{\pi} + \frac{A}{\pi} \left(\frac{\pi^2}{6} - \frac{1}{2} \right) = \frac{A}{2}$$