Study Material

B Sc General

PHYSICS

Semester-IV

DSC-1D

Duration- 1 hr

Date: 06/04/2020

Time- 8AM-9AM

Topic: Fourier's theorem

* ফুরিয়ারের উপপাদ্যের সাহায্যে $f(x) = x^2$ সমীকরণটি বিশ্লেষণ

Ex-1 কর। যেখানে $0 \le x \le 2\pi$

$\Rightarrow$ ফুরিয়ারের উপপাদ্য থেকে আমরা জানি,

$$f(x) = a_0 + \sum_{n=1}^{\alpha} (a_n \cos nx + b_n \sin nx)$$

এখন $a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(x)\, dx$

$$= \frac{1}{2\pi} \int_0^{2\pi} x^2\, dx = \frac{1}{2\pi} \left[\frac{x^3}{3}\right]_0^{2\pi}$$

$$= \frac{4\pi^2}{3}$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx\, dx$$

$$= \frac{1}{\pi} \int_0^{2\pi} x^2 \cos nx\, dx$$

$$= \frac{1}{\pi} \left[x^2 \frac{\sin nx}{n} \Big|_0^{2\pi} - \frac{2}{n} \int_0^{2\pi} x \sin nx\, dx \right]$$

$$= -\frac{2}{\pi n} \left[-\frac{x \cos nx}{n} + \frac{\sin nx}{n^2} \right]_0^{2\pi}$$

$$= \frac{2}{\pi n^2} [2\pi] = \frac{4}{n^2}$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} y \sin nx\, dx$$

$$= \frac{1}{\pi} \int_0^{2\pi} x^2 \sin nx\, dx$$

$$= \frac{1}{\pi} \left[-\frac{x^2 \cos nx}{n} \Big|_0^{2\pi} + \frac{2}{n} \int_0^{2\pi} x \cos nx\, dx \right]$$

$$= \frac{1}{\pi} \left[-\frac{4\pi^2}{n} \cos 2n\pi + \frac{2}{n} \left\{ \frac{x \sin nx}{n} + \frac{\cos nx}{n^2} \right\}_0^{2\pi} \right]$$

$$= \frac{1}{\pi} \left[-\frac{4\pi^2}{n} + \frac{2}{n}(0) + \frac{2}{n^3} (\cos 2n\pi - 1) \right]$$

$$= -\frac{4\pi}{n}$$

সুতরাং ফুরিয়ার (মূল) বিস্তৃতিকে লেখা হল

$$f(x) = \frac{4\pi^2}{3} + 4\sum_{n=1}^{\alpha} \frac{\cos nx}{n^2} - 4\pi \sum_{n=1}^{\alpha} \frac{\sin nx}{n} \quad \text{—} (1)$$

<u>অনুসিদ্ধান্ত I</u>: (1) নং সমীকরণের উভয়পক্ষে $x = \pi$ বসিয়ে পাওয়া যায়

$$\pi^2 = \frac{4\pi^2}{3} + 4\left(-\frac{1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} - \cdots\right) - 0$$

$$\left(\text{যেহেতু } f(x) = x^2 \text{ তাই } f(\pi) = \pi^2\right).$$

$$\therefore \pi^2 - \frac{4\pi^2}{3} = 4\left(-\frac{1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} - \cdots\right)$$

$$\therefore -\frac{\pi^2}{3} = 4\left(-\frac{1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} - \cdots\right)$$

$$\therefore \frac{\pi^2}{12} = \left(\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots\right) \quad \text{—} (2)$$

<u>অনুসিদ্ধান্ত II</u>: (1) নং সমীকরণের উভয় পক্ষে $x = \pi/2$ বসিয়ে পাওয়া যায়

$$\left(\frac{\pi}{2}\right)^2 = \frac{4\pi^2}{3} + 0 - 4\pi\left(\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right)$$

$$\therefore \frac{\pi}{4} = \frac{4\pi}{3} - 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right)$$

$$\therefore 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right) = \pi\left(\frac{4}{3} - \frac{1}{4}\right)$$

$$\therefore 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{13}{48}\pi \quad \text{—} (3)$$